Taking species abundance distributions beyond individuals

Abstract

The species abundance distribution (SAD) is one of the few universal patterns in ecology. Research on this fundamental distribution has primarily focused on the study of numerical counts, irrespective of the traits of individuals. Here we show that considering a set of Generalized Species Abundance Distributions (GSADs) encompassing several abundance measures, such as numerical abundance, biomass and resource use, can provide novel insights into the structure of ecological communities and the forces that organize them. We use a taxonomically diverse combination of macroecological data sets to investigate the similarities and differences between GSADs. We then use probability theory to explore, under parsimonious assumptions, theoretical linkages among them. Our study suggests that examining different GSADs simultaneously in natural systems may help with assessing determinants of community structure. Broadening SADs to encompass multiple abundance measures opens novel perspectives in biodiversity research and warrants future empirical and theoretical developments.

Keywords

Species abundance distribution, body-size, size distribution, size–density relationship, biomass, energy use, size–energy relationship, resource partitioning, macroecology, metabolic theory.

INTRODUCTION

The species abundance distribution (SAD) describes the commonness and rarity of species in ecological communities. The SAD is one of the oldest patterns in ecology (Fisher et al. 1943; Preston 1948; MacArthur 1957), and remains at the core of ecological research (McGill et al. 2007; Alonso et al. 2008; Dornelas & Connolly 2008). Repeatable patterns in the shape of the SAD across taxa and environments (the hollow curve, with few dominant and many rare species) suggest that
this distribution reflects common underlying principles that structure ecological communities. The SAD is widely used to test alternative hypotheses regarding the determinants of community structure and biodiversity (Caswell 1976; Hubbell 2001; Etienne & Oll 2005; Chave et al. 2006; McGill et al. 2006a).

Most work on the SAD has focused on the frequency distribution of the number of individuals per species (numerical abundance). This is a natural way to characterize SADs and predictions for this type of abundance distribution are available for a variety of models. Numerical abundance is commonly used in community ecology as a measure of the local dominance or success of a species (Preston 1962). The use of numerical abundance also has a precedence in population biology as the most natural measure of demography (Volterra 1926). In evolutionary biology, it is an important determinant of the genetic variability in populations, influencing the relative strength of selection and genetic drift (Wright 1938). However, the number of individuals is only one way to characterize the relative abundance of species within communities, and there are several reasons why abundance has also been measured differently. Abundance has been measured as the biomass and/or resource use of species populations in systems ecology, in order to better understand compartmentalization and fluxes of carbon, nutrients and energy in ecosystems (Odum 1983). Combined measures of abundance and biomass have also been used by marine biologists as a diagnostic tool (Warwick 1986). Numerical abundance, biomass, resource use and their interrelationship have been the main focus of empirical and theoretical macroecology, to understand the division of food and space among species (Brown & Maurer 1989; Thibault et al. 2004; White et al. 2004; Connolly et al. 2005; Harte et al. 2008). Finally, for pragmatic reasons (e.g. modularity in plants) field ecologists have sometimes used percent cover, point quadrat frequency or biomass as proxies for numerical abundance (Wilson et al. 1996; Chiarucci et al. 1999; Tokeshi 1999; Latimer et al. 2005).

Patterns observed for different abundance measures are expected to be similar if numerical abundance, biomass and resource use are proportional to one another. This condition may be achieved if interspecific differences are negligible, or if all individuals can be considered to be ecologically equivalent (MacArthur 1960; Hubbell 2001). For example, ecologists have often viewed the ‘classical’ SAD (constructed from numerical abundance) as representing the outcome of rules governing how species partition resources (Miomura 1932; MacArthur 1957, 1960; Sugihara 1980; Tokeshi 1990, 1999; Marquet et al. 2003). For this characterization to be valid, it is necessary to assume that individuals across species are comparable in their body-size and resource use, as originally noted by MacArthur (1960):

“It is assumed that the species whose abundances are being compared are roughly of same size, so that an individual of one species is comparable to an individual of any of the others.”

This assumption is likely violated in natural systems for several reasons. First, within and between communities, species’ body-sizes typically vary by several orders of magnitude (Brown & Nicoletto 1991), and large-bodied species generally use more energy than small-bodied ones (Kleiber 1947; Peters 1983). Second, the most numerous species do not always contain the most biomass or use the most resources, as they sometimes contain many small individuals (Brown & Maurer 1986; Maurer & Brown 1988). Third, individuals of the same size can have a very different resource use, as a consequence of differences in traits such as internal nutrient concentrations (Sterner & Elser 2002) or life histories (Bielby et al. 2007). Finally, it has been increasingly realized that individuals within populations of the same species can strongly vary in size, due in part to age structure, with important consequences for ecological interactions (Werner & Gilliam 1984; Kooijman 2000). Hence, different abundance measures are not necessarily well correlated (Saint-Germain et al. 2007), and SADs constructed from different abundance measures may well exhibit fundamental differences. Partly for this reason, discussion on the use of SADs has drifted away from the perspective of resource partitioning and ecosystem functioning.

It has recently been suggested that investigating abundance measures other than the number of individuals may lead more directly to mechanistic theory in ecology, placing alternative currencies at the front line of future SAD research (McGill et al. 2007). Three fundamental questions follow naturally: (i) What are the similarities and differences in the way individuals, biomass and resource use are distributed among species? (ii) What are the constraints that link the distributions constructed from different abundance measures? (iii) What insights can be gained from exploring the distribution of biomass and resource use among species? Aspects of these questions generated a suite of papers with equivocal conclusions (Harvey 1987; Sugihara 1989; Page et al. 1991; Taper & Marquet 1996). In these papers, the authors focused on comparing the variance between the distributions of numerical abundance, biomass and resource use, and drew upon two classical allometries (relationships of the form $c x^{a}$): the allometry between population density and body-size (Damuth 1981), and between body-size and energy use (as a proxy for resource use, Kleiber 1947). Some of the results from this body of work suggested that the equitability with which species divide individuals, biomass or resource use is similar (Sugihara 1989; Taper & Marquet 1996) while others suggested important differences (Harvey 1987; Tokeshi 1990; Page et al. 1991). Since then, there has been continued focus on species numerical abundance.
distributions, but few papers have examined SADs in parallel using alternative measures for abundance (but see Thibault et al. 2004; Connolly et al. 2005; Saint-Germain et al. 2007).

In this paper we generalize the ‘classical’ SAD based on numerical abundance into a set of Generalized Species Abundance Distributions (GSADs) that encompass alternative measures, specifically biomass and resource use. We start with an empirical analysis of GSADs for four major taxonomic groups at various spatial scales. This empirical analysis aims to test the equivalence between GSADs, and to search for general patterns in the way GSADs differ from one another. We then develop, under parsimonious assumptions, a conversion framework that links GSADs to one another. This theoretical analysis provides the first step towards a theoretical understanding of differences between GSADs. We illustrate the utility of our theoretical framework for interpreting the data, and discuss the implications of our findings for our understanding of community structure. Finally, we highlight future developments that are needed to fully understand the interrelationships between GSADs.

**EMPIRICAL EVIDENCE FOR MAJOR DIFFERENCES BETWEEN GSADs**

We compare GSADs within a wide variety of taxonomic groups and habitats, by comparing their variance, skewness and spatial scaling. We use the terminology ‘species individuals distribution’ (SID) to refer to the ‘classical’ SAD (the one constructed from numerical abundance), while SBD and SED refer to the species biomass and energy distributions respectively (Table 1).

### Data and methods

We use data from a total of 1533 communities spanning four major taxonomic groups: birds, fish, small mammals and trees. The bird data is from the Breeding Bird Survey of North America (BBS; \(n=1400\); Robbins et al. 1986; Sauer et al. 2008), the fish data from an island scale survey of Trinidad \((n=76);\) Magurran & Phillip 2001), the small mammal data from the Sevilleta LTER (Ernest et al. 2000) and the Portal Project \((n=7);\) Brown 1998; Ernest et al. in press), and the tree data from a 50-ha tropical forest plot at Barro Colorado Island (BCI; \(n=50\) plots of 1 ha; Condit 1998). Each bird species is assigned an average mass from Dunning (1993). The population biomass of each fish species comes from a direct measurement in the field. The population biomass of each mammal species is obtained from summation of individual body-size measurements. Individual tree biomass is estimated from an empirical interspecific allometry based on diameter at breast height (Brown 1997), providing a first approximation of more specific allometries incorporating wood density (Chave et al. 2003).

Empirical data on the energy use (a proxy for resource use) of each species within sampled communities are typically unavailable. We estimate per-capita energy use for birds and mammals based on published allometries for field metabolic rates for the respective groups (Nagy et al. 1999). For fish and trees, we use published allometries for basal metabolic rates (Gillooly et al. 2001). A detailed description and treatment of the data sources can be found in Appendix S1. In evaluating the results, the reader should note that while numerical abundance and biomass are either

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**Table 1 Measurements, their corresponding distributions and notations**

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Symbol</th>
<th>Macroecological distribution</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalized SADs (GSADs)</td>
<td>Number of individuals: measured in a fixed area (density)</td>
<td>(N)</td>
<td>Species individuals distribution SID</td>
</tr>
<tr>
<td></td>
<td>Biomass: sum of individual masses over a species’ population</td>
<td>(M)</td>
<td>Species biomass distribution SBD</td>
</tr>
<tr>
<td></td>
<td>Energy use: sum of individual energy use over a species’ population (reflects resource utilization)</td>
<td>(E)</td>
<td>Species energy distribution SED</td>
</tr>
<tr>
<td>Other macro ecological distributions</td>
<td>Body-size: average body-mass over a species’ population</td>
<td>(\bar{m})</td>
<td>Species size distribution SsD</td>
</tr>
<tr>
<td></td>
<td>Individual energy use: average individual energy use over a species’ population (reflects individual resource use)</td>
<td>(\bar{i})</td>
<td>Species individual energy distribution SeD</td>
</tr>
</tbody>
</table>

We denote abundance measures by upper-case symbols (e.g. \(N, M\) and \(E\)) and measures at the individual level (per-capita measurements) by lower-case symbols (e.g. \(m\) and \(i\)). Overlines stand for averages over individuals of a given species (e.g. \(\bar{m}\) and \(\bar{i}\)). The ‘classical’ species abundance distribution (the one constructed from the number of individuals) is coined ‘species individuals distribution’ (SID), while the term ‘generalized species abundance distributions’ (GSADs) is used to refer to any of the distributions of numerical abundance (SID), biomass (SBD) or energy use (SED).

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measured directly or estimated with confidence, energy use is necessarily calculated by making assumptions about the form of the relationship between metabolic rate and body size. We used the best available estimations of energy distributions given the nature of our data.

We compare the SID, SBD and SED using three simple characterizations: (i) the variance of log-transformed abundance values, a measure directly related to many common evenness metrics (e.g. Evar; May 1975; Smith & Wilson 1996), and used in previous comparisons of GSADs (Harvey 1987; Sugihara 1989; Pagel et al. 1991; Taper & Marquet 1996), (ii) the skewness of log-transformed abundance values (e.g. lognormal distributions have no skew), a measure of asymmetry in a frequency distribution based on the third central moment, widely used to characterize SADs (e.g. Nee et al. 1991; Gregory 1994; Hubbell 2001; Magurran 2003; McGill 2003) and (iii) visual evaluation of the variation of the shape of the distributions with spatial scale (i.e. unveiling; Preston 1962; Nee et al. 1991; Connolly et al. 2005). Using the log-transformed data focuses on the shape of the distribution as usually characterized by ecologists (e.g. the famous ‘Preston plots’; Preston 1962) and eliminates any dependence on measurement units. In combination these three characterizations provide a broad and general picture of the similarities and differences between species abundance distributions (SADs) based on different measures.

**How GSADs differ from one another**

We compare for each local community within each data set the variance of log-transformed abundance measures against one another (Fig. 1, left panels, see also details and statistics in Appendix S2). Under the null hypothesis that GSADs are equivalent, the resulting data points would cluster around the 1 : 1 line. Rather than this null result, the data points consistently lay above the 1 : 1 line, demonstrating that the partitioning of individuals, biomass and energy is not equivalent. Overall, SIDs are more evenly distributed than SBDs and SEDs, and SEDs more evenly distributed than SBDs. This general pattern occurs across all the data sets, though it is less consistent in the fish data.

We compare for each local community within each data set the skewness of log-transformed abundance measures against one another (Fig. 1, right panels and Appendix S2). Under the null hypothesis that GSADs are equivalent, the resulting data points would cluster around the 1 : 1 line. Deviations from this null assumption are less consistent among data sets than the results concerning variance. The mammal communities appear to have equivalent skewness regardless of the abundance measure. The other three data sets do tend to exhibit changes in the average sign and/or magnitude of the skewness among the different abundance measures. The bird data do not show a clear tendency for the direction of skew of the SID (there are about as many sites with positively skewed distributions as with negatively skewed).

**Figure 1** How diversity distributions constructed from different abundance measures differ from one another. Comparison of the variance (left panels) and skewness (right panels) between GSADs. Each data point represents a local community. Black triangles: birds; $n = 1400$. Blue squares: fish; $n = 76$. Red circles: mammals; $n = 7$. Green crosses: trees; $n = 50$. The data points would fall on the black 1 : 1 line if GSADs were equivalent with respect to variance and skewness. A Var–Var plot between two different abundance measures (left panels) illustrates which of the two is the most evenly distributed between species. Top left: individuals are overall more evenly distributed than biomass (statistically significant in all data sets); middle left: individuals are overall more evenly distributed than energy use (significant in all data sets except the fish); bottom left: energy use is overall more evenly distributed than biomass (significant in all data sets). The skewness–skewness plots (right panels) show that, except for the mammals, skewness is not equivalent among abundance measures. Detailed results and statistics are provided in Appendix S2. Two outliers from the fish data were excluded from the results in the left panels to allow for clearer presentation (Outlier 1: $\text{Var} \log(N) = 1.0$, $\text{Var} \log(E) = 20.3$, $\text{Var} \log(M) = 48.5$; Outlier 2: $\text{Var} \log(N) = 1.7$, $\text{Var} \log(E) = 4.0$, $\text{Var} \log(M) = 15.3$).
skewed distributions), while the SBD and SED are generally positively skewed. The fish data show the opposite pattern with more communities having positively skewed SIDs, and no clear tendency in the skewness of the other GSADs. The plants show the most extreme differences, cleary shifting from positively skewed SIDs to negatively skewed SBDs and SEDs. In sum, with the exception of the mammal data, all data sets exhibit differences in skewness among different GSADs, but with no clear general pattern.

We compare within each data set the global spatial scaling of GSADs. In general, the shape of sampled SADs differs from that of the underlying community SAD (Green & Plotkin 2007). In particular, SADs often appear truncated when the rarest species are not sampled (veil effect; Preston 1962). Recently, Connolly et al. (2005) showed that the SID unveils at much larger spatial scales than the SBD. In other words, non-truncated SBDs are obtained at spatial scales where SIDs are still mostly truncated. Our data suggest that what Connolly et al. refer to as a ‘scale discrepancy’, a phenomenon in which the SID and the SBD unveil at different spatial scales, is not specific to the coral systems they studied. Indeed, this phenomenon also occurs in each data set with multiple scales of observation (i.e. all the systems except the mammals; Fig. 2), and also holds for the SED (although in the case of the fish data the unveiling is less rapid for energy than for biomass). This phenomenon provides further support for fundamental differences between GSADs.

A THEORETICAL EXPLORATION OF THE LINK BETWEEN GSADs

The empirical results on variance, skewness and unveiling presented above demonstrate clear deviations from the null hypothesis that GSADs are equivalent. These results also show general patterns in the difference between GSADs. Understanding the origin of deviations between GSADs in natural communities will likely provide clues about the processes underlying their structure. Here, we provide the first steps of a conversion framework to analytically link GSADs to one another. Developing such a framework necessarily involves making assumptions about the way different abundance measures relate to one another. After presenting general conversions between the shape and moments of distributions under these assumptions, we evaluate their accuracy in predicting our data.

Conversion framework

We aim for generality and investigate the conversion between any two diversity distributions (probability density function describing the frequency of species with a given attribute) $S_X$ and $S_Y$ constructed from any log-transformed measurements $X$ and $Y$. In the context of converting GSADs, $X$ and $Y$ may be replaced by log$(N)$, log$(M)$ or log$(E)$ (see Empirical Evaluation below). The conversion between two diversity distributions $S_X$ and $S_Y$ is given by basic probability theory:

$$S_Y(Y) = \int P(Y|X)S_X(X)dX$$

(1)

$P(Y|X)$ is the probability that a species with measurement $X$ also has measurement $Y$. For simplicity, we use the integral notation for both continuous and discrete $X$, but the integral should be interpreted as a sum when $X$ is discrete. If $X$ is independent of $Y$, or in other words if $X$ has no explanatory power on $Y$, $P(Y|X)$ is $P(Y)$ and no relationship between $S_X$ and $S_Y$ can be derived. However, abundance measures are not independent (Figs S1 and S2). We thus assume that $X$ has an explanatory power on $Y$, and write:

$$Y = f(X) + Z$$

(2)

where $Z$ is a random variable capturing the deviation of $Y$ from its functional dependency ($f$) on $X$. By writing this equation, we implicitly assume that ecological and evolutionary processes act primarily on $X$, and that $Y$ is determined, with some variation, by $X$. We further assume that $Z$...
is independent of $X$. These assumptions may be violated in natural communities, and should be considered as first parsimonious approximations.

In the particular case where $X$ is the main explanatory variable for $Y, Z$ can be thought of as a residual error independent of $X$, usually normally distributed. Denoting $P_X$ the probability density distribution of this error, it is straightforward to show (Appendix S3) that the distribution $S_Y$ is related to the distribution $S_X$ by:

$$S_Y(Y) = \int P_X(Y - f(X)) S_X(X) dX$$ (3)

This general equation may be used to derive relationships between $S_X$ and $S_Y$ for any specific form of the distributions of $X$ and $Y$, the functional relationship between them, and the form of the residual error, provided that the error is independent of $X$. Table 2 shows specific results when the variation $Z$ around the relationships between $X$ and $Y$ is either ignored or normal (i.e. log-normal on a linear scale), and when $f$ is either unspecified or linear, which is the case for a power-law allometry between variables (see Appendix S3 for details).

There is evidence for the existence of power-laws in nature. In particular, empirical and theoretical work support that individual energy use scales with body-size as approximately $e \propto m^a$ (size–energy relationship; Kleiber 1947; West et al. 1997) and that in some cases population density scales with body-size as $N \propto m^{-b}$ (size–density relationship; Damuth 1981; Peters 1983; White et al. 2007). When these relationships are assumed to be power-law, the relationship between any of $N, M$ and $E$ is also expected to be power-law (Appendix S4). Power-laws with log-normal error (corresponding to linear relationships with normal error for logarithmically transformed variables) are also the simplest and most natural approximations describing the relationships between numerical abundance, biomass and the energy use of species. By substituting two of $\log(N), \log(M)$ or $\log(E)$ in $Y$ and $X$, the formulas in Table 2 show that GSADs are interrelated patterns rather than independent ones. These formulas also show that a priori, the shape of GSADs is not conserved: resource division rules predicting a log-normal SBD (Sugihara 1980) may well produce a non log-normal SID.

### Converting the variance and skewness of GSADs

Many studies are more interested in simple characterizations of the distributions based on their higher moments, such as the variance and skewness, than in their detailed shape. Here, we investigate the relationship between the variance and skewness of GSADs. We consider the case of power-law allometries, as they constitute convenient approximations. On a log-log scale, a power-law allometry with exponent $a_{Y|X}$ where $X$ is the explanatory variable reads:

$$Y = \log(a_{Y|X}) + a_{Y|X}X + Z_{Y|X}$$ (4)

where $a_{Y|X}$ is a normalization constant. Below, we assume as above that $Z_{Y|X}$ is independent of $X$. The relationship between the variance of $X$ and $Y$ is straightforward:

$$\text{Var}[Y] = a_{Y|X}^2 \text{Var}[X] + \text{Var}[Z_{Y|X}]$$ (5)

Note that here the variance of $X$ and $Y$ are not (measurement) errors of a single value of $X$ and $Y$ for a single species. Rather, $\text{Var}[X]$ and $\text{Var}[Y]$ pertain to the overall variance in observed $X$ and $Y$ values across species. When applied to the SID, SBD and SED, this relationship may be used to examine under which conditions biomass and energy use are expected to be more or less equally distributed than individuals (Appendix S4).

### Table 2 Conversion formulas

<table>
<thead>
<tr>
<th>No error</th>
<th>Log-normally distributed error</th>
</tr>
</thead>
<tbody>
<tr>
<td>General allometry</td>
<td>$S_Y(Y) = \frac{S_X(f^{-1}(Y))}{</td>
</tr>
<tr>
<td>Power-law allometry</td>
<td>$S_Y(Y) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(Y - \log(x))^2}{2\sigma^2}$</td>
</tr>
</tbody>
</table>

Formulas providing the conversion from any macroecological distribution $S_X$ (constructed from measurement $X$) to any macroecological distribution $S_Y$ (constructed from measurement $Y$). We assume that $X$ is an explanatory variable for $Y$, that $Y$ is functionally linked to $X$ through the relationship $f$, and that the deviation $Z$ from this functional relationship is independent of $X$. $X$ and $Y$ are typically log-transformed data. On a log-log scale, a power-law allometry between $X$ and $Y$, with exponent $a_{Y|X}$ reads $Y = \log(a_{Y|X}) + a_{Y|X}X + Z_{Y|X}$ where $a_{Y|X}$ is a normalization constant, i.e. $f$ is linear ($f(x) = \log(\epsilon) + ax$). For example, $a_{M|N} = \frac{1}{\gamma} (1 + \gamma a_{N|X})$ and $a_{E|X} = \gamma (a_{E|X})$ where $a_{N|X}$ is the power-law exponent of the size–density relationship (Damuth 1981; White et al. 2007), and $a_{E|X}$ is the power-law exponent of the relationship between individual energy use and body-size (Kleiber 1947; Peters 1983; Appendix S3).

In the absence of error about the relationships between measurements, these conversions correspond to a simple change of variable.
The skewness of a variable $X$ is given by:

$$\text{Skew}[X] = \frac{E[(X - E[X])^3]}{\text{Var}[X]^{3/2}}$$  \hspace{1cm} (6)

The relationship between the skewness of $X$ and $Y$, for any centred and unskewed random variable $Y_{Y|X}$ (e.g. normally distributed on a log-scale; see Appendix S3 for the more general case of skewed error), is given by:

$$\text{Skew}[Y] = \frac{a_{Y|X}^3 E[(X - E[X])^3]}{(a_{Y|X}^2 \text{Var}[X] + \text{Var}[Y_{Y|X}])^{3/2}}$$  \hspace{1cm} (7)

This relationship shows that the direction of the skewness of distributions may change if the power-law exponent $a_{Y|X}$ is negative. The amplitude of the skewness is expected to be conserved for a perfect allometry between $X$ and $Y$ ($Z_{Y|X} = 0$), but to decrease from $S_X$ to $S_Y$ in the more realistic case when the allometry is not perfect.

Equations 5 and 7 provide the relationships between the second and third moments of $S_X$ and $S_Y$, under the hypothesis that $X$ is an explanatory variable for $Y$. Assuming instead that $Y$ is an explanatory variable for $X$, the relationships will be different, as a result of error propagation (Taper & Marquet 1996; Appendix S3). These relationships may thus be used to evaluate, in a given system, if data are most consistent with a model where $X$ or $Y$ is considered to be an explanatory variable for the other. This possible application is illustrated below.

**Illustration and empirical evaluation**

We illustrate the developments above by focusing on the relationship between $\text{Var}[\log M]$ and $\text{Var}[\log N]$; we test the ability of Equation 5 to predict patterns in the empirical data, and discuss their potential application for deciphering processes driving community assembly. We note that the biomass of a species is directly given by $M = N \bar{m}$, where $\bar{m}$ is the average body-size of the species’ population.

To derive the relationship between $\text{Var}[\log M]$ and $\text{Var}[\log N]$, we need to assume, as noted above, that either $M$ or $N$ is an explanatory variable for the other. In natural systems, complex ecological interactions act in concert to determine the biomass and numerical abundance of species. It is likely that neither $M$ nor $N$ is a direct explanatory variable for $N$, nor $N$ is a direct explanatory variable for $M$, but rather that processes act both on $N$ and $M$ in a correlative manner (Taper & Marquet 1996). However, we ask the following questions: are our data consistent with the hypothesis that community structure is primarily driven by processes such as resource division rules (e.g. Marquet et al. 2003), acting on the relative biomass of species more directly than on their relative population size? In this case, $M$ is the natural explanatory variable, constraining population sizes and the demographic processes from which they result. Alternatively, are our data consistent with the hypothesis that community structure is primarily driven by demographic processes (e.g. Maurer 1999), acting on the numerical abundance of species more directly than on their biomass? In this case, $N$ is the natural explanatory variable, constraining the biomass of species.

To answer these questions, we consider two simplified scenarios used as benchmarks for more realistic models. In the first scenario, ecological processes determine the variance in the SBD, and we aim to predict the variance in the SID. We test two alternative hypotheses: either the body-size of species has no effect on their ability to accumulate biomass (e.g. large species have small population sizes and no advantage in resource monopolization), or the body-size of species influences their ability to accumulate biomass (e.g. large species have an advantage in resource monopolization). In the later case, we assume that the dependency of biomass on body-size is power-law (exponent $a_{M|m}$). The resulting relationship between the variances, obtained with the approach outlined in the Conversion Framework, is given in Table 3 (formulas in the top table, Appendix S3). In the second scenario, we assume that ecological processes determine the variance in the SID, and we aim to predict the variance in the SBD. We test two alternative hypotheses: either the body-size of species has no effect on their numerical abundance (i.e. $N$ is independent of $\bar{m}$), or the body-size of species influences demographic processes, and thus ultimately numerical abundance (e.g. small species have shorter generation times and reach higher densities). In the later case, we assume that the dependency of numerical abundance on body-size (i.e. the size–density relationship; Damuth 1981; White et al. 2007) is power-law (exponent $a_{N|m}$). The predicted relationships between the variance corresponding to each scenario are given in Table 3 (formulas in the bottom table; Appendix S3).

We test the ability of each scenario to reproduce the data. When body-size is assumed to have an effect on either numerical abundance or biomass, the power-law exponent of the corresponding relationship is estimated by regression in each local community. We compare observed and predicted values for the variance (Fig. 3), and test the performance of the various predictions by evaluating their accuracy, as measured by the standardized root mean squared error (Table 3; predictions with best accuracy are the ones with lowest values). The results suggest that our data are best described when the numerical abundance of species is considered to be the primary explanatory variable in the system, and species’ biomass results from an attribution of numerical abundance across species independent of their body-size. The second best prediction is obtained when the relative biomass of species is considered.

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to be the primary explanatory variable, and their numerical abundance results from an attribution of biomass across species dependent on their body-size (i.e. body-size influences the capacity of species to accumulate biomass). The third best prediction is obtained when the relative biomass of species is considered to be the primary explanatory variable, and their numerical abundance results from an attribution of biomass across species independent of their body-size. Finally, considering the numerical abundance of species as the primary explanatory variable, with a dependency of species’ capacity to accumulate individuals on their body-size, yields particularly unrealistic predictions. This simple analysis suggests that in our systems, the main effect of the body-size of a species is to affect its ability to accumulate biomass rather than to influence population density via the production or recruitment of new offspring.

**SYNTHESIS AND FUTURE DIRECTIONS**

**Differences between GSADs**

Models of SADs based on resource division rules often assume that individuals, biomass or resource use are interchangeable:

“A niche translates ultimately into numbers of organisms (or biomass)” (Sugihara 1980).

The question of the interchangeability between abundance measures was raised over 20 years ago (Harvey 1987), generating a decade long debate (Sugihara 1989; Pagel et al. 1991; Taper & Marquet 1996) that ended in the following terms (Taper & Marquet 1996):

“The apportionment of individuals and resources, among species, are equivalent processes.”

Our results contradict this statement. Below we discuss the cause and consequence of differences in the variance, skewness, and unveiling of GSADs.

The first difference between GSADs observed in our data is their variance (Fig. 1). This result differs from the result of Taper & Marquet (1996) due to the difference in our approach to analyzing empirical data. The authors estimated the empirical variance in energy among species using a theoretical relationship linking variances in energy and numerical abundance. As they note, their estimation likely suffers from error propagation. In our empirical analyses, we directly estimate the energy use of each species from fitted relationships between field or basal metabolic rate and body-size, and calculate the variance among species a posteriori. Because we have direct data on organismal body-size (except for the birds, although the error introduced by using a mean species mass is expected to be small), we eliminate the principal source of error in the theory-based variance prediction. One limitation of our approach is that
we ignore the error introduced by using the size–energy allometry. However, including error propagation would increase the variance in energy-use, given the biologically realistic assumption that metabolic rate is determined by body-size (‘body-size causal model’). This suggests that the disparity in variances between the SED and the SID is likely to be even greater than we observed. Furthermore, as our analyses of biomass and numerical abundance are based on direct field measurements, the disparity in variances between the SBD and the SID is unequivocal.

In the four taxonomic groups analysed, biomass and resources are overall less equitably distributed among species than are individuals. Intuitively, small-bodied species live below the density for which they would receive an equal share of the total biomass or resources, while large-bodied species live above this density. In other words, large species have an advantage in resource monopolization, an idea sometimes summarized as ‘bigger is better’ (Brown & Maurer 1986; Maurer & Brown 1988; Pagel et al. 1991; McGill 2008). These observations go against the idea embedded in the so-called ‘energetic equivalence rule’ (Damuth 1981). Nearly 30 years ago, Damuth (1981) gathered data showing that the numerical abundance of a species decreases as a $-\frac{1}{2}$ power of its body-size, which, combined with the $\gamma$ power-law relationship between body-size and metabolic rate, suggests that the resource use of a species is independent of its population density and body-size. Under this rule, one would expect the variance in energy among species to be approximately constant with respect to the variance in numerical abundance. While our estimation of energy use is to be taken with caution, our analyses strongly suggest that the energy equivalence rule is not achieved in our data sets, where variance in energy and numerical abundance are highly correlated. These results are consistent with the observation that the energetic equivalence rule is rarely supported within local communities (Blackburn & Gaston 1997; White et al. 2007).

The second difference between GSADs observed in our data is their skewness (Fig. 1). Early statistical explanations for the origin of the shape of the SAD (May 1975) were largely based upon the normal (unskewed) nature of the SAD (plotted on a log-scale). More recent developments have been motivated by the repeated observation that SADs tend to exhibit a significant left skew on a logarithmic scale (Nee et al. 1991; Gregory 1994; Hubbell 2001; Magurran 2003; McGill 2003). Our results suggest that future theories could profitably focus on why different abundance measures have different degrees of skew.

The third difference between GSADs observed in our data is the scale at which they unveil (Fig. 2). Biomass and energy distributions unveil at a much smaller spatial scale than SIDs. These results are consistent with patterns observed by Connolly et al. (2005) in coral and fish communities. Connolly et al. (2005), basing their justification on Taper & Marquet (1996), suggested that the differences in unveiling between the SAD and SBD provide support against niche-based origins of the SAD. Our results suggest that SID and SBD are not expected to be similar, irrespective of which ecological processes generate them. The difference in the spatial scale at which SID and SBD unveil appears to be a general phenomenon, and also applies to the energy distribution.

The rapid unveiling of biomass and resource use suggests that GSADs based on these measures may avoid a number of problems associated with the distribution of numerical abundance. The number of individuals or areas sampled in typical ecological surveys is often too small to yield completely unveiled SIDs, particularly in species-rich systems, such as tropical or microbial communities. Recon-
Linkages between GSADs

Linking the SID to other important macroecological distributions is fundamental to the quest towards ‘unified’ theories that can explain many macroecological patterns in concert (Hubbell 2001; McGill et al. 2007; Harte et al. 2008; Storch et al. 2008), while it is well established that the SID is linked to spatial biodiversity patterns (e.g. the species area and distance-decay relationships) through aggregated sampling (He & Legendre 2002; Plotkin & Muller-Landau 2002; Morlon et al. 2008), we show here a series of linkages of a different nature. Linkages between the SID and (non-spatial) macroecological distributions (in particular the SBD and SED) arise from relationships between body-size, energy use and numerical abundance, and may be derived using the formulas presented in Table 2. These results reinforce the vision of the SID as ‘the master pattern’, from which many others may be derived (McGill et al. 2007).

Our results suggest that the shape of GSADs is not generally conserved when converting from one abundance measure to another, so that a theoretical prediction for one abundance measure is not directly applicable to another. In particular, theory relying on resource partitioning assumptions (e.g. MacArthur 1957, 1960; Sugihara 1980; Tokeshi 1990, 1999) will only directly predict the shape of SEDs. To the extent to which abundance is related to body-size or energy use, the conversion from one GSAD to another may be approached using the general formulas presented in Table 2. Such conversions can potentially extend the breadth of current ecological theory from one to multiple GSADs. From a practical standpoint, such conversions may be used to assess the shape of GSADs that are difficult to measure empirically. For example, our conversion framework can be used for predicting SEDs, which are relevant to resource-based models but often empirically intractable. This conversion framework could also provide a tool for estimating the classical SAD in systems where this is currently challenging, for example in microbial systems.

The relationships between the variance and skewness of GSADs depend on two major factors: which abundance measure is considered to be an explanatory variable, and the nature of the dependency between species’ traits (e.g. body-size) and abundance (Table 3 and Appendix S3). These findings support and extend the pioneer work by Taper & Marquet (1996) showing that the expected relationship between the variance of GSADs depends on what the authors called ‘models of causality’. We further suggest to leverage this dependency to identify which of the models is most consistent with empirical data. When data are available simultaneously for different abundance measures in a given system, we can indeed test the relative accuracy of predictions derived under different scenarios (Fig. 3 and Table 3).

When applying these analyses to our data, we find that the predictions with better accuracy are given by a ‘N causal’ model, from which biomass is deduced from independent pairing between species population size and body-size (i.e. body-size has no net effect on the demography of species), or a ‘M causal’ model, from which numerical abundance is deduced from directional pairing between species biomass and body-size (i.e. body-size influences the ability of species to accumulate biomass). Interestingly, while the ‘N causal’ model coupled with a dependency of population size on species body-size is the first model one would think about given Damuth’s rule (Damuth 1981), this model yields particularly poor predictions (Fig. 3 bottom right panel; see also Taper & Marquet 1996; their fig. 2a). This result is not surprising given the weak dependence of population size on body-size in our data (results not shown), and would probably be different in communities showing a clear size-density relationship (e.g. Schmid et al. 2000). Drawing inference about causality from relationships between the variance and skewness of distributions offers promising avenues; however, further empirical and theoretical research is needed to extend the approach, and to fully understand its promise and limitations.

Future directions

We have described empirical patterns in the comparison between GSADs, and theoretical approaches for understanding these patterns. These analyses constitute a preliminary investigation in an area of research that clearly needs further development.

Collecting data encompassing various abundance measures

Comparing distributions constructed from various abundance measures can provide insights into pattern and process in ecological communities. However, data sets encompassing several measures are scarce, and those containing direct information on resource use are almost nonexistent. Future empirical work on SADs will greatly benefit from simultaneously collecting measures of numerical abundance, biomass and resource use.

Relaxing assumptions in the conversion framework

We based our conversion framework on the simplest reasonable set of assumptions. We assumed that $X$ is an
explanatory variable for $Y$ (or inversely), and that there exists a functional relationship between $X$ and $Y$. However, two variables depending on one another do not necessarily do so directly, as the dependence can also be driven by a third variable influencing both of them simultaneously (Taper & Marquet 1996). It is not clear how distinguishing between direct causal pathways vs. joint dependence will change the results, a question that will require future work.

We assumed that the deviation $Z$ about the functional relationship between two variables can be considered as independent of the explanatory variable. In the case of converting between numerical abundance and biomass, this hypothesis is equivalent to assuming that the deviation about the size–density relationship is independent of body-size. This hypothesis is well supported in some data (e.g. Schmid et al. 2000), but violated in others (e.g. Marquet et al. 1995).

We did not consider intraspecific variation in our framework. Intraspecific variation in body-size is not expected to have any effect on the conversion between the SID and the SBD, as the relationship between the biomass and numerical abundance of a species only involves its average body-size ($M = N \overline{m}$ is exact, by definition of $\overline{m}$). However, an exact conversion between the SID (or SBD) and the SED will involve writing the average individual energy use of a species as a function of its body-size distribution (frequency distribution of individual body-sizes within species).

**Integrating sampling theory within the conversion framework**

Recent theoretical developments based on sampling theory have advanced our understanding of the spatial scaling of the SID (Green & Plotkin 2007). It appears that species biomass and energy distributions unveil at different spatial scales than SID do (our Fig. 2; see also Connolly et al. 2005). While this phenomenon has potentially important theoretical and practical implications (Connolly et al. 2005), it is not well understood. Coupling a sampling framework with the conversion formulas presented in this paper will likely allow for a better understanding of the different spatial scaling perceived by various GSADs.

**Towards a mechanistic modelling of GSADs**

Given the complexity of ecological processes, statistically based approaches for modelling SADs receive a renewed interest (e.g. Green & Plotkin 2007; Pueyo et al. 2007; Harte et al. 2008). The theoretical part of this paper is purely statistical. For example, it does not depend on whether species are considered to be ecologically equivalent or idiosyncratic, or whether they occupy similar or different trophic levels or niches. We have shown, extending the work of others (Taper & Marquet 1996), that even a purely statistical conversion between GSADs does depend on assumed ‘models of causality’. While this dependence may be used to provide insights into ecological processes, more robust tests of theories will necessitate a mechanistic modelling of the SBD and SED along with that of the SID (e.g. O’Dwyer et al. 2009).

**Linking GSADs to other macroecological patterns**

We focused in this paper on the relationships between GSADs. However, the generality of the probability theory we use offers broader perspectives for linking frequency distributions in ecology. The allometries between numerical abundance, body-size and metabolic rate allow linking GSADs to two other macroecological patterns: the well-known species size distribution (SsD; Hutchinson & MacArthur 1959; Brown & Maurer 1989; Etienne & Olff 2004; Allen et al. 2006; Clauset & Erwin 2008), and the newly introduced species individual energy distribution (SeD; Appendix S3, Figs S1 and S2). Loehle (2006) noted previously that the SID and SsD are related to one another through the size–density relationship. He showed that the geometric SID can result from a uniform species-size distribution and an exact power-law size–density relationship. This result is easily reproduced using our framework (with $Y = N$ and $X = \overline{m}$; Appendix S3), and the sensitivity of this result to deviations from the exact power-law size–density relationship could further be investigated using our framework.

Realizing that the SID and SsD are linked raises the question of which of the two patterns we should try to find mechanisms or models for. Both the SID and SsD have been the subject of much attention in the literature, and many models have been developed to explain their shape (Hutchinson & MacArthur 1959; Brown & Maurer 1989; Etienne & Olff 2004; Allen et al. 2006; Clauset & Erwin 2008). However, the SID and SsD have been studied in isolation. Applying the general formulas provided in this paper with specific shapes for the species individuals or size distributions will help bridge this gap.

In an even broader context, frequency distributions are not limited to the GSADs, SsD and SeD. Frequency distributions of traits other than body-size and individual-energy use are commonly used. Ecologists look at species trait distributions (Reich et al. 2003), trait-trait allometries (Wright et al. 2004) and abundance-trait relationships McGill et al. (2006a,b). The mathematical framework we use is relevant to linking any two GSADs, any GSAD to any species trait distribution (given a mathematical link between traits and abundance), or any two mathematically related traits (Table 2; Stegen & White 2008). These relationships add to the growing literature on integrating patterns in ecology (Reuman et al. 2008).
ACKNOWLEDGEMENTS

This manuscript is the product of a working group on ‘Tools and fresh approaches for species abundance distributions’ sponsored by the National Center for Ecological Analysis and Synthesis and led by B.J.M. We gratefully acknowledge the funding and logistical support provided by NCEAS. E.P.W. led the sub-working group resulting in this paper. The authors recognize that contributing to this paper was one of the key occupations of R.S.E. while he was a Courtesy Research Associate with J.L.G. and H.M.) at the University of Oregon. H.M. acknowledges support from the University of Oregon. Various individual authors acknowledge support from NSF, NCEAS, NWO-ALW and GAAVCR. We thank Pablo Marquet for discussions of the ideas, as well as Jérôme Chave and three anonymous referees for suggestions that improved our manuscript. We are thankful to Jamie Gillow for providing us with his data on basal metabolic rates. We thank Mike Friggens and the Sevilleta LTER for providing access to their small mammal data and Morgan Ernest and James Brown for providing the data from the Portal LTREB. Data collection for these sites is funded by NSF. We are grateful to Dawn Phillip for collecting the Trinidadian fish data and the Darwin Initiative (UK) for funding the survey. We also acknowledge the many volunteers as well as USGS and CWS researchers for collecting and making available the Breeding Bird Survey data. The BCI forest dynamics research project was made possible by National Science Foundation grants to Stephen P. Hubbell, support from the Center for Tropical Forest Science, the Smithsonian Tropical Research Institute, the John D. and Catherine T. MacArthur Foundation, the Mellon Foundation, the Celera Foundation and numerous private individuals, and through the hard work of over 100 people from 10 countries over the past two decades. Finally, we thank members and visitors of the Bohannan Lab and Green Lab at the University of Oregon for their comments, in particular Albert Barberan, Brendan Bohannan, Kathryn Docherty, James O’Dwyer, Becky Mueller and Liz Perry.

REFERENCES


**SUPPORTING INFORMATION**

Additional Supporting Information may be found in the online version of this article:

**Figure S1** General link between diversity distributions.

**Figure S2** Macroecological space.

**Figure S3** Equitability in the distribution of individuals, biomass and energy.

**Appendix S1** Detailed data sources and methods.

**Appendix S2** Detailed empirical results.

**Appendix S3** Detailed derivations.

**Appendix S4** Equitability in the distribution of individuals, biomass and energy.

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Editor, Jérôme Chave
Manuscript received 9 December 2008
First decision made 11 January 2009
Manuscript accepted 19 March 2009